Fast recovery of rotational symmetry parameters using gradient orientation

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Abstract. Symmetry detection is important in the area of computer vision. A simple and fast algorithm for the recovery of rotational symmetry parameters is developed. The fold number of a rotationally symmetrical shape is obtained from the gradient orientation histogram of the input gray-level image using the Fourier method, based on the relationship between the periodicity of the orientation histogram and the fold number of the symmetrical shape. It takes only 0.06 s CPU time for a 256 $\times$256 gray image on a Sun SPARC10. Both simulated and real images are tested and the results are very convincing. [S0091-3286(97)01504-3]

Subject terms: rotational symmetry; orientation histogram; fold number; gradient orientation; Fourier transform.

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1 Introduction

Many objects around us are strongly constrained. For instance, not only cultural artifacts but also many natural objects are rotationally symmetrical. The goal of an image understanding system is mainly to identify and locate a specified object in the scene. In such cases, the system must have some knowledge of the shape of the desired object. Symmetries are good candidates for describing shape, which is a powerful concept that facilitates object detection and recognition in many situations. These representations can be used in robotics for recognition, inspection, grasping, and reasoning.

Symmetry can be defined in terms of three transformations in n-dimensional Euclidean space E^n: reflection, rotation, and translation. Formally, a subset S of E^n is symmetrical with respect to a transformation T if $T(S)=S$. We concentrate only on rotational symmetry in this paper. A rotationally symmetrical shape is said to be n-fold symmetrical if the shape overlaps with itself after being rotated by any multiple of $2\pi/n$ around its center.

Most of the work carried out on rotational symmetry detection has been based on edge, contour, or point set information. Burton et al.[1] considered a simple indexing scheme to implement the exponential pyramid data structure for particular symmetries (the rotational symmetries they considered are only 90 and 180 deg rotational). Wolter et al.[2] described exact algorithms for detecting all rotational and involutional symmetries in point sets, polygons, and polyhedra. Highnam[3] presented optimal algorithms for finding rotational symmetries of a planar point set. Zabrodsky et al.[4] defined a continuous symmetry measure to quantify the symmetry of objects. They also presented a multiresolution scheme that hierarchically detects symmetrical and almost symmetrical patterns in Ref. 5. But no result on rotational symmetry was given. Yuen[6] and Yuen and Chan[7] used the Hough transform to detect skewed and rotational symmetry on a set of points. Yip et al.[8] also used a Hough transform technique to detect rotational symmetry of line drawings. Jiang and Bunke presented an algorithm for determining rotational symmetries of polyhedral objects[9]. Parry-Barwick and Bowyer[10] developed methods that can detect both hierarchical and partial symmetry of 2-D set-theoretic models with components constructed with a few straight edges or polynomials. This method has the disadvantage of being computationally intensive. Masuda et al.[11] described a method of extracting rotational and reflectional symmetry by performing a correlation with the rotated and reflected images. To extract rotational symmetry, they took the original image and then applied all possible rotations to it and correlated in two dimensions with the original image. The transformation was carried out using each pixel as the center of rotation in turn. A good match signified rotational symmetry. The disadvantages were associated with the high computational cost and memory requirements.

Tsai and Chou[12] and Chou et al.[13] have developed a series of methods for solving the recognition problems concerning rotationally symmetrical shapes. Their methods do not seem efficient, since the fold number must be found beforehand. Pei and Lin use a modified Fourier descriptor to normalize rotationally symmetrical shapes that can detect the fold number and rotation angle simultaneously on a binary image.[14] Leou and Tsai[15] proposed a simple but effective algorithm to determine the rotational symmetry of a given closed-curve shape.

In this paper, we investigate the use of the statistics of the gradient orientation for rotational symmetry detection in a gray-scale image. The next section briefly describes the gradient of an image and its orientation histogram. Then the algorithm for finding the fold number and the separation lines for the symmetry (or the orientation of the object) are given. The algorithm is tested on both simulated and real images.

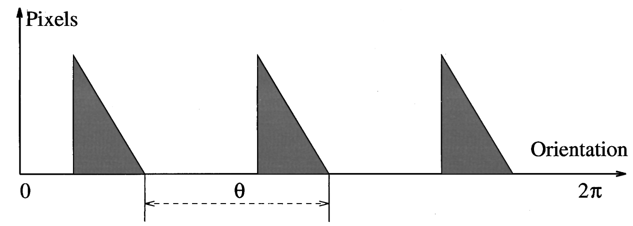


Fig. 1 Illustrative shape of the gradient orientation histogram for a rotationally symmetrical object.

2 Orientation Histogram

For an image $I(x,y)$, the surface gradient vector $[I_x, I_y]^T$ is defined by $I_x = \partial I(x,y)/\partial x$, $I_y = \partial I(x,y)/\partial y$. The orientation of this gradient vector can be calculated by $\phi = \tan^{-1}[I_y/I_x]$. The domain of ϕ is $[0,2\pi)$. The gradient is obtained by using a 5×5 Sobel filtering operation.[16] It splits the kernels into 2×1 subkernels and iteratively calculates the responses. The result is two responses for the x and y directions, respectively. Then the gradient orientation at this point of the image can be obtained. When both I_x and I_y are zero at a point in the image, the gradient at this point is treated as not defined. By having a set of bins in the range of [0,360), the orientation histogram of the image can therefore be obtained. We note that the histogram shape should be translationally symmetrical or nearly translationally symmetrical because of digitization error. Figure 1 illustrates the gradient orientation histogram for a rotationally symmetrical object. In the figure, θ is the fold angle (in this case, it is 120 deg or $2\pi/3$). The fold number equals $2\pi/\theta$.

3 Fold Number of Rotational Symmetry

Our algorithm is based on the intensity gradient orientation distribution of the image. From the previous section, the histogram of this gradient orientation image can be obtained. It is clear that this histogram function is periodic with period 360 deg (or 2π). That is, $h(j) = h(j \pm 2k\pi)$, $k = 0,1,2,\ldots$, and $0 \leqslant j < N$, where $h(j)$ is the gradient orientation histogram (in our case $N = 360$). In particular, 360 bins is very useful because if n is the fold number, n divides 360 for $n = 2,3,4,5,6,8,9,10,12$. For a rotationally symmetrical object, the orientation histogram should have circularly repetitive patterns. We can choose to evaluate the following function for obtaining the fold number:

$$c_{\text{mean}}(n) = \frac{1}{n} \sum_{k=1}^{n} \sum_{j=0}^{N-1} h(j) h\left(\frac{kN}{n} + j\right), \tag{1}$$

where n is the candidate fold number of symmetry $(n \geqslant 2)$. If for a particular n, $c_{\text{mean}}(n)$ produces a maximum value, we take this n as the fold number.

For a rotationally symmetrical object, its gradient orientation histogram $h(j)$ will have subperiods of N. Since $h(j)$ is periodic, its Fourier transform $F_h(u)$ as defined in Eq. (2) is useful for detecting subperiods:

$$F_h(u) = \sum_{j=0}^{N-1} h(j) \exp(-2\pi iju/N), \quad u = 0,1,\ldots,N-1. \tag{2}$$

The Fourier spectrum $|F_h(u)|^2 = F_h(u) F_h^*(u)$ is useful for finding the base frequency, and hence the subperiod and this subperiod is related to the fold number. Instead of using Eq. (1), we will search the sum of the subset of the Fourier spectrum because of the relationship stated in the following proposition.

Proposition

$$n^{-1} \sum_{k=1}^{n} c(kL) = N^{-2} \sum_{v=0}^{L-1} |F_h(vn)|^2, \tag{3}$$

where

$$c(j) = N^{-1} \sum_{k=0}^{N-1} h(k) h(k+j),$$

and L is the fold angle $(N = nL)$.

We use the right-hand side of the Eq. (3) as rewritten in Eq. (4) to find the fold number. When a particular n gives a large sum of the Fourier spectrum over multiple frequencies of n, we take this n as the fold number:

$$\sum_{v=0}^{L-1} |F_h(vn)|^2, \quad n \geqslant 2. \tag{4}$$

The algorithm we use for finding the fold number of a rotationally symmetrical object is:

1. Obtain the orientation histogram of the input grayscale image.
2. Calculate the Fourier transform and power spectrum of the orientation histogram.
3. Search for the largest summation, as in Eq. (4) when $n \geqslant 2$. For practical applications, this n can be further constrained by $n \leqslant K$, where K is the largest possible fold number, say 15.

4 Orientations of Shapes

After the fold number of an object has been obtained, it is also necessary to determine the orientation of the object. This can be done in many ways. Lin et al. use two types of shape specific points, called the fold-invariant centroid (FIC) and the fold-invariant radius-weighted mean (FIRWM), for detecting the orientations of rotationally symmetrical shapes.[17] Chou et al.[13] introduced a method, called the fold principal axis, to define the orientations of rotationally symmetrical shapes. Lin[18] described a universal principal axis method to define shape orientation.

In this paper, the orientation of the object is easily obtained by evaluating the phase of the Fourier coefficient at the corresponding fold number. The exponential form of $F_h(u)$ can be written as $F_h(u) = |F_h(u)| \exp[j\phi_h(u)]$, where $|F_h(u)|$ is the magnitude function, and $\phi_h(u)$ is its phase angle. This u is the fold number obtained in the previous section. The phase angle corresponding to the fold angle is the orientation of the symmetrical object. The sequence of operations for obtaining the rotational symmetry information are:

1. Obtain the gradient of the input image.
2. Calculate the histogram of the gradient orientation.
3. Obtain the fold number of the object, as described in the previous section.
4. Obtain the orientation of the object using Fourier phase information.
5. Draw lines from the center of mass to show the symmetry parameters.

5 Experimental Results

The results for the described algorithm on simulated and real images are given in this section. No initial smoothing was applied to the original image before the gradient operation. Because of digitization effects, the boundary of an object with high contrast consists of mostly zigzagged short line segments, and very often these line segments are either horizontal or vertical with diagonal segments connecting the horizontal or vertical short lines. Therefore, in most of the gradient orientation histograms, peaks often appear at 45 deg and its multiples (90, 135, 180, 225, 270, and 315 deg). The histogram is circularly smoothed using a median filter of length 5, that is, the smoothing window is wrapped around at the ends since the angular data are circularly continuous.

The histogram property is a necessary but not a sufficient condition for symmetry detection; that is, for certain nonsymmetrical objects, its orientation histogram might still be translationally symmetrical. One example of this is shown in Fig. 2. Figure 2(a) shows a nonsymmetrical object, while Fig. 2(b) displays its idealized orientation histogram. Although the orientation histogram is translationally symmetrical, the object does not have the rotational symmetry property. However, as long as we know that the object in the image is rotationally symmetrical, we can apply the algorithm to detect rotational symmetry. If the condition is not sufficient, i.e., when the orientation histogram shows translational symmetry but actually the object is not rotationally symmetrical, a further step might be necessary to check whether the object is actually rotationally symmetrical. These parameters or symmetry hypotheses can be verified by obtaining a symmetry measure (see Ref. 4). If this symmetry measure gives a low value, we say that the object is not symmetrical. If an object is convex, the histogram property will not only be a necessary, but also a sufficient condition for symmetry detection.

Figure 3 shows the process of obtaining the symmetry information. Figure 4 gives the result of the symmetry detection algorithm applied to several simulated images.

The algorithm works not only for binary images, where the gradient information mainly occurs along the boundaries of objects, but also for gray-level images, where the gradient operation is performed on every pixel in the image.

Figure 5 shows the results of detecting the rotational symmetries of some gray-scale images. There are two dark strips on the triangle of Fig. 5(b), and they will to some

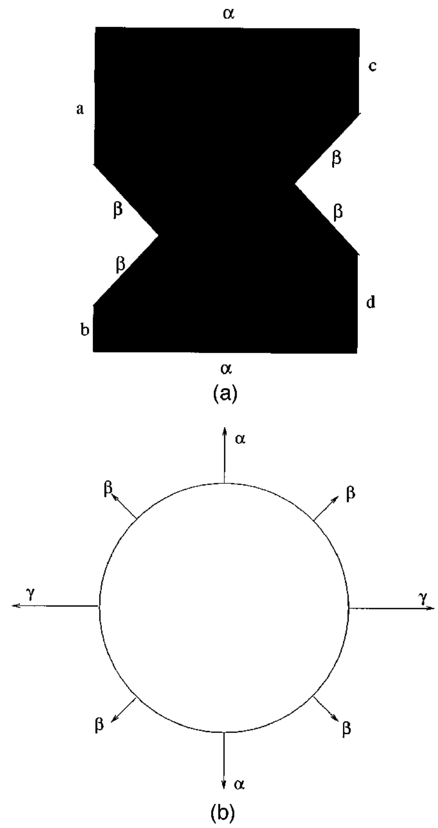


Fig. 2 (a) Counterexample when the algorithm might not work and (b) ideal orientation histogram for (a) (where $\gamma = a + b = c + d$).

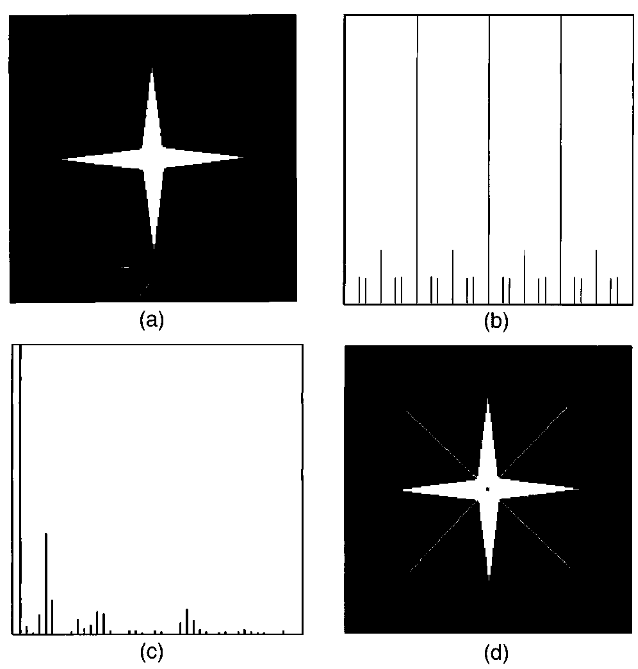


Fig. 3 Symmetry detection process: (a) original image, (b) the gradient orientation histogram of image (a), (c) magnitude of the Fourier transform of the gradient orientation histogram (b) (zero frequency was suppressed for display purpose), and (d) the symmetry orientation obtained.

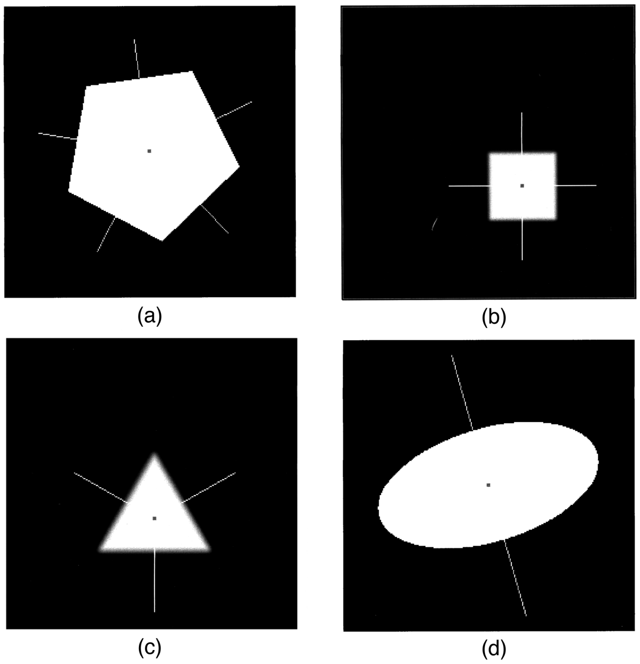


Fig. 4 Symmetry detection results for simulated images. The white lines show the symmetry information of a shape: (a) a pentagon, (b) a square, (c) a triangle, and (d) an ellipse.

extent affect the shape of the orientation histogram. However, the algorithm has still detected the correct symmetry. Figure 6 shows the results for images to which noise has been added. Figure 7(a) shows an example of symmetry detection using a range image of a propeller. Although the object in the image is not exactly rotationally symmetrical (the leaf in the lower part of the image is smaller than the other two), the algorithm can still pick up the most likely symmetry fold. Figures 7(b) and 7(c) show the results of

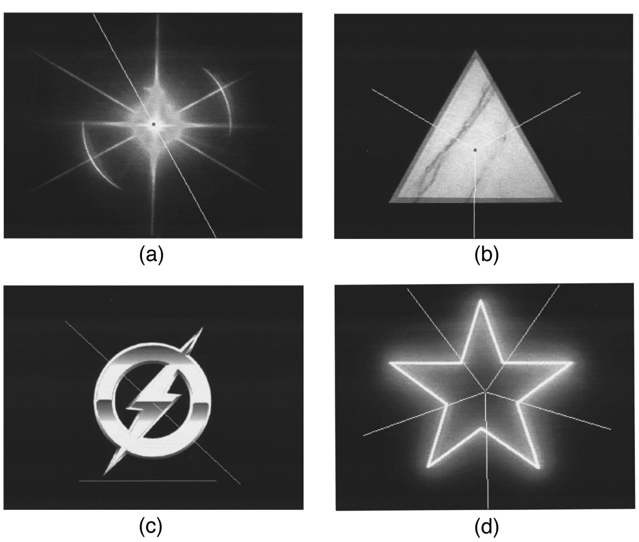


Fig. 5 Symmetry detection for some more complex images (with the symmetry information overlaid): (a) a star shaped image, (b) a more complex triangle image, (c) a two-fold symmetrical object, and (d) a five-fold star shape.

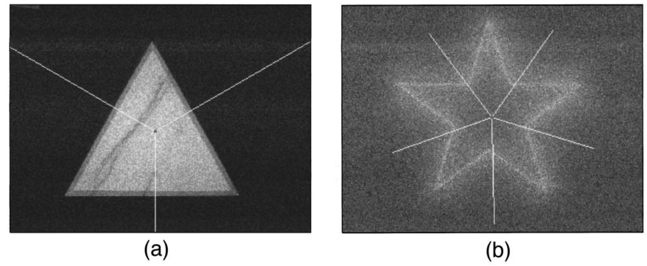


Fig. 6 Symmetry detection for images with noise added: (a) noisy image of Fig. 5(b), and (b) noisy image of Fig. 5(d).

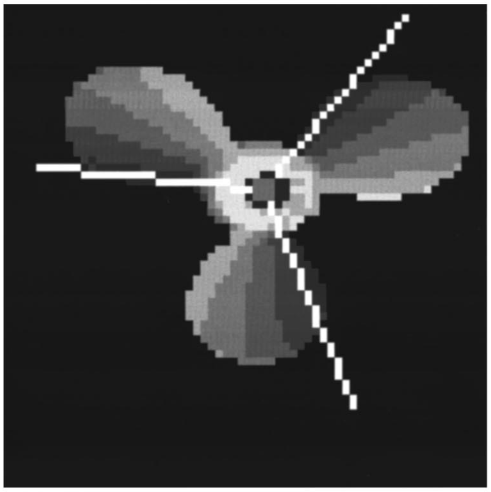
the algorithm applied to two other real images. The typical CPU running time for a 256×256 image is 0.06 s on a Sun SPARC10. The program has not been optimized to increase the speed.

For objects that are not rotationally symmetrical, there will be a trivial symmetry; that is, if an object is rotated 360 deg, it will come back to the original position. The fold number in this case will be 1. If the background is textured, the gradient orientation input of the background may mess up the algorithm. In this case, a prior segmentation of the object from the background will be necessary to ensure that the algorithm works. After the foreground or object has been isolated from the textured background, the described symmetry detection algorithm can be applied to the region that the object occupies. The segmentation of objects in a textured background is itself a research topic, and we do not deal with it here.

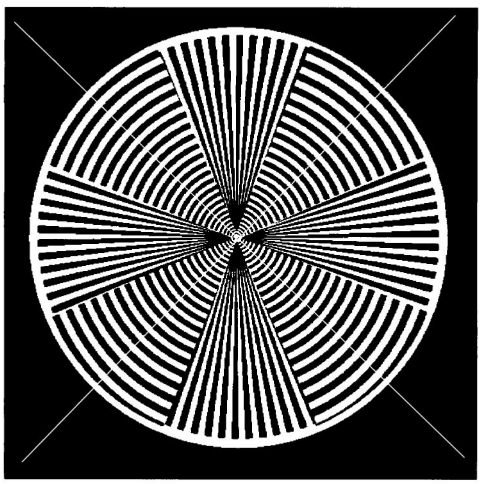
6 Conclusions

A simple and fast rotational symmetry detection algorithm has been developed that employs only the original gray-level image and the gradient orientation information. The results show that the statistics of the gradient orientation can be used to obtain the rotational symmetry of an object in a gray-level image. The CPU time for a 256×256 image takes only about 0.06 s on a Sun SPARC10, and most of this time is spent on the gradient operation. The Fourier transform of the 1-D orientation histogram is very fast. Both simulated and real images have been tested and the results are very convincing. The information concerning the rotationally symmetrical shape can be of use for many applications (such as robot operation, further image segmentation, etc).

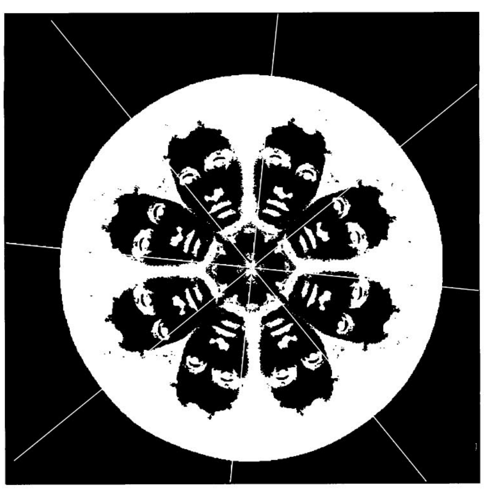
Work has been performed only on a single object in an image. The same algorithm can be applied to the separated multiple objects in an image that can be obtained by segmentation. For every single object, apply the algorithm described in this paper to obtain the rotational symmetry information. From the nature of the algorithm, only ''low-level'' processing is involved, and therefore, it is possible to implement the algorithm in parallel.



(a)



(b)



(c)

Fig. 7 Symmetry detection results for real images. The white lines separate different folds of a rotationally symmetrical shape: (a) a range image of a propeller, (b) fluctuating crosses, and (c) a kaleidoscopic portrait of the inventor of the kaleidoscope, David Brewster. (The last two pictures are reproduced with kind permission from Ref. 19, p. 144, Figs. 2.14 and 2.70.)

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References

1. F. W. Burton, J. G. Kollias, and N. A. Alexandridis, ''An implementation of the exponential pyramid data structure with application to determination of symmetries in pictures,'' *Comp. Vis. Graph. Image Process.* **25**, 218–225 (1984).
2. J. D. Wolter, T. C. Woo, and R. A. Volz, ''Optimal algorithms for symmetry detection in two and three dimensions,'' *Vis. Comput.* **1**, 37–48 (1985).
3. P. T. Highnam, ''Optimal algorithms for finding the symmetries of a planar point set,'' *Inf. Proc. Lett.* **22**, 219–222 (1986).
4. H. Zabrodsky, S. Peleg, and D. Avnir, ''A measure of symmetry based on shape similarity,'' in *IEEE Proc. Computer Vision and Pattern Recognition*, pp. 703–706 (1992).
5. H. Zabrodsky, S. Peleg, and D. Avnir, ''Hierarchical symmetry,'' in *Proc. Int. Conf. on Pattern Recognition*, Vol. 3, pp. 9–11 (1992).
6. S. Y. K. Yuen, ''Shape from contour using symmetries,'' in *IEEE Proc. Eur. Conf. on Computer Vision*, pp. 437–453, Springer-Verlag, Lecture Notes in Computer Science No. 427 (1990).
7. K. S. Y. Yuen and W. W. Chan, ''Two methods for detecting symmetries,'' *Pattern Recog. Lett.* **15**, 279–286 (1994).
8. R. K. K. Yip, W. C. Y. Lam, P. K. S. Tam, and D. N. K. Leung, ''A Hough transform technique for the detection of rotational symmetry,'' *Pattern Recog. Lett.* **15**, 919–928 (1994).
9. X. Y. Jiang and H. Bunke, ''A simple and efficient algorithm for determining the symmetries of polyhedra,'' *Comput. Vis. Graph. Image Process. Graph. Models Image Process.* **54**, 91–95 (1992).
10. S. Parry-Barwick and A. Bowyer, ''Symmetry analysis and geometric modelling,'' in *Proc. Digital Image Computing: Techniques and Applications*, K. K. Fung and A. Ginige, Eds., Vol. 1, pp. 39–46, Australian Pattern Recognition Society, Sydney, Australia (1993).
11. T. Masuda, K. Yamamoto, and H. Yamada, ''Detection of partial symmetry using correlation with rotated-reflected images,'' *Pattern Recog.* **26**, 1245–1253 (1993).
12. W.-H. Tsai and S.-L. Chou, ''Detection of generalized principal axis in rotationally symmetric shapes,'' *Pattern Recog.* **24**, 95–104 (1991).
13. S.-L. Chou, J.-C. Lin, and W.-H. Tsai, ''Fold principal axis—a new tool for defining the orientation of rotationally symmetric shapes,'' *Pattern Recog. Lett.* **12**, 109–115 (1991).
14. S.-C. Pei and C.-N. Lin, ''Normalization of rotationally symmetric shapes for pattern recognition,'' *Pattern Recog.* **25**, 913–920 (1992).
15. J.-J. Leou and W.-H. Tsai, ''Automatic rotational symmetry determination for shape analysis,'' *Pattern Recog.* **20**, 571–582 (1987).
16. P. E. Danielsson and O. Seger, ''Generalized and separable Sobel operators,'' in *Machine Vision for Three-Dimensional Scenes*, H. Freeman, Ed., pp. 347–379, Academic Press, New York (1990).
17. J.-C. Lin, S.-L. Chou, and W.-H. Tsai, ''Detection of rotationally symmetric shape orientations by fold-invariant shape-specific points,'' *Pattern Recog.* **25**, 473–482 (1992).
18. J.-C. Lin, ''Universal principal axes: an easy-to-construct tool useful in defining shape orientations for almost every kind of shape,'' *Pattern Recog.* **26**, 485–493 (1993).
19. N. Wade, *Visual Allusions: Pictures of Perception*, Lawrence Erlbaum, Hove and London (1990).

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