

# TRINOCULAR STEREO IMAGE RECTIFICATION IN CLOSED-FORM ONLY USING FUNDAMENTAL MATRICES

*Changming Sun*

CSIRO Mathematics, Informatics and Statistics, Locked Bag 17, North Ryde, NSW 1670, Australia  
changming.sun@csiro.au

## ABSTRACT

Trinocular stereo image rectification is a process to transform a set of three images into a new set so that the epipolar lines in the transformed images have the same direction as the image row or column and matching epipolar lines in different images have the same row or column indices to enable an efficient and reliable dense stereo matching. In this paper we propose a new closed-form method to rectify three-view stereo images just using fundamental matrices. The algorithm involves only direct and purely geometric transformation processes. No iteration or optimization process is involved in our method. Real images have been used for testing purposes, and accurate results have been obtained using our algorithm.

**Index Terms**— Trinocular stereo images, rectification, closed-form, fundamental matrix, epipole

## 1. INTRODUCTION

Stereo image rectification is an important step for three dimensional scene analyzes. The rectification process transforms the input images into new ones with epipolar lines being along the horizontal and vertical axes and matching epipolar lines from different images having the same row or column indices. For stereo matching, image rectification can increase both the reliability and the speed of disparity estimation. Stereo rectification usually requires calibrated camera parameters or parameters of uncalibrated cameras in the form of fundamental matrices; and most algorithms require the initial feature matching points as part of inputs.

There are articles on two-view image rectification using known calibrated camera parameters [1, 2]. There are also a large number of algorithms developed for uncalibrated two-view cases. Some of these methods use a direct sampling approach based on the epipolar geometry [3, 4, 5]. Some algorithms require the fundamental matrix and matching points information to obtain the rectifying matrices [6, 7, 8]. Another class of algorithms for two-view cases are the estimation of the rectifying matrices directly from image matching

points [9, 10]. Two-view rectification methods without using the image matching points but only using the fundamental matrix are also developed in [11, 12, 13, 14, 15].

Below are researches on three-view image rectification. Ayache and Hansen presented a technique for calibrating and rectifying a pair or triplet of images with estimated camera matrix [16]. A similar method for efficient rectification for trinocular stereo vision was given in [17]. Shao and Fraser also developed a rectification method for calibrated trinocular cameras [18]. Laganière and Kangni presented a projective rectification method for image triplets [19]. Zhang et al. proposed a linear method for trinocular rectification for accurate stereoscopic matching [20]. Sun presented several methods, such as rotation and skew, affine transformation, and vanishing points methods, for rectifying the reference image and then applying matching transformations for rectifying the rest of the image triplets [21].

In this paper, we propose a new closed-form algorithm for rectifying three uncalibrated images when only a pair of fundamental matrices are used and no iterative parameter optimization step is involved. All the steps that we use only involve geometric or perspective transformation. Because our algorithm only uses the fundamental matrices for rectification, it is especially useful when the fundamental matrices are estimated without the use of matching points as in [22], or when the fundamental matrices are obtained from a trilinear tensor which can be obtained from matching lines, not points.

## 2. RECTIFYING TRINOCULAR IMAGES

### 2.1. Obtaining Fundamental Matrices

With trinocular images, it has been shown in [23] that the point correspondence constraint among the three views is expressed by the trilinear tensor. It has also been shown that the fundamental matrices which govern the epipolar geometry between any two views of the three can be obtained from the trilinear tensor. The fundamental matrices can also be estimated directly from image matching points between any two views of the three [24]. Once the fundamental matrices among the three views are available, they can then be used for rectification purposes.

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## 2.2. Projection Matrices for the Reference Image

With the three images in the image triplet, we use “left”, “right” and “top” to represent each image. The left image is also the reference image. Given the epipolar geometry defined by the fundamental matrix  $\mathbf{F}_{12}$  between the left and right images, a pair of epipoles  $\mathbf{e}_{12}$  and  $\mathbf{e}_{21}$  in these two images can be obtained by solving  $\mathbf{F}_{12}\mathbf{e}_{12} = 0$  and  $\mathbf{e}_{21}^T\mathbf{F}_{12} = 0$  using a singular value decomposition method. Similarly with  $\mathbf{F}_{13}$  for the left and top images, a pair of epipoles  $\mathbf{e}_{13}$  and  $\mathbf{e}_{31}$  can also be obtained.  $\mathbf{e}_{12}$  and  $\mathbf{e}_{13}$  are in the left image, while  $\mathbf{e}_{21}$  is in the right image and  $\mathbf{e}_{31}$  is in the top image. These epipoles will be transformed to  $(1, 0, 0)^T$  (on the  $x$ -axis) or  $(0, 1, 0)^T$  (on the  $y$ -axis) after rectification.

Next, we will present one algorithm for rectifying the reference image as described in [21]. Assuming that the image center is at  $(u, v, 1)^T$  for all the three images, one can use the following transformation  $\mathbf{T}$  to shift the origin of the image coordinate system to the image center:

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & -u \\ 0 & 1 & -v \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{R}_1 = \begin{pmatrix} \cos \theta_1 & \sin \theta_1 & 0 \\ -\sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then the image can be rotated such that the epipole after translation  $\mathbf{e}'_{12} = \mathbf{T}\mathbf{e}_{12} = (e'_{12}[0], e'_{12}[1], 1)^T$  is further moved onto the  $x$ -axis. This rotation transformation takes the form  $\mathbf{R}_1$  with  $\theta_1 = \arctan(e'_{12}[1]/e'_{12}[0])$ .

After the epipole  $\mathbf{e}_{12}$  has been transformed to lie on the  $x$ -axis, we need to move epipole  $\mathbf{e}_{13}$  to the  $y$ -axis. This can be obtained by a skew operation:

$$\mathbf{S} = \begin{pmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where  $s = -e''_{13}[0]/e''_{13}[1]$  with  $(e''_{13}[0], e''_{13}[1], 1)^T = \mathbf{R}_1\mathbf{T}\mathbf{e}_{13}$ . Now the two epipoles  $\mathbf{e}_{12}$  and  $\mathbf{e}_{13}$  have been transformed onto the image axes with value  $k_x$  on the  $x$ -axis for the first epipole and value  $k_y$  on the  $y$ -axis for the second epipole. The next step will be to shift the epipole positions to infinity. This transformation can be achieved using the following matrix:

$$\mathbf{K} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/k_x & -1/k_y & 1 \end{pmatrix} = \mathbf{K}_x\mathbf{K}_y$$

with

$$\mathbf{K}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/k_x & 0 & 1 \end{pmatrix}, \quad \mathbf{K}_y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/k_y & 1 \end{pmatrix}$$

The combined transformation matrix for the reference image is:

$$\mathbf{P}_1 = \mathbf{KSR}_1\mathbf{T}.$$

## 2.3. Shifting Epipoles for Right and Top Images

The transformation matrix for the right image can take the form  $\mathbf{P}_2 = \mathbf{K}_2\mathbf{R}_2\mathbf{T}$ , generating horizontal epipolar lines. For the top image, transformation matrix  $\mathbf{P}_3 = \mathbf{K}_3\mathbf{R}_3\mathbf{T}$  can

be used. The  $\mathbf{K}_2$  and  $\mathbf{K}_3$  matrices for the right and top images are

$$\mathbf{K}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/k_2 & 0 & 1 \end{pmatrix}, \quad \mathbf{K}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/k_3 & 1 \end{pmatrix}$$

and  $\mathbf{R}_2$  for the right image can be obtained similarly as  $\mathbf{R}_1$ .  $\mathbf{R}_3$  takes the same form as  $\mathbf{R}_1$  but with the rotational angle  $\theta_3 = \arctan(e'_{31}[0]/e'_{31}[1])$  with  $\mathbf{e}'_{31} = \mathbf{T}\mathbf{e}_{31}$ . The  $k_2 = e''_{21}[0]$  with  $\mathbf{e}''_{21} = \mathbf{R}_2\mathbf{T}\mathbf{e}_{21}$  and  $k_3 = e''_{31}[1]$  with  $\mathbf{e}''_{31} = \mathbf{R}_3\mathbf{e}'_{31}$ . Applying  $\mathbf{P}_3$  to the top image generates vertical epipolar lines.

## 2.4. Aligning Matching Epipolar Lines

After applying  $\mathbf{P}_1$ ,  $\mathbf{P}_2$ , and  $\mathbf{P}_3$  to the left, right and top images respectively, epipolar lines become horizontal and vertical. But the matching epipolar lines are not aligned yet. The alignment of the matching horizontal epipolar lines between the left and right images can be carried out similarly as in the two-view case of [15], but using  $\mathbf{P}_{1x} = \mathbf{K}_x\mathbf{SR}_1\mathbf{T}$  (rather than  $\mathbf{P}_1$ ) and  $\mathbf{P}_2$ . The alignment transformation is  $\mathbf{A}_2\mathbf{T}_v$  for the right image.

We now design a transformation matrix which aligns the matching vertical epipolar lines between the left and top images. Similar to the case for aligning horizontal epipolar lines [15], three regularly spaced points along the horizontal line passing through the center of the left image can be used. The matching epipolar lines can be obtained from the fundamental matrix between the left and top images. We use the following  $\mathbf{T}_u$  to align the middle vertical epipolar lines:

$$\mathbf{T}_u = \begin{pmatrix} 1 & 0 & -u_m \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{A}_3 = \begin{pmatrix} w_x & 0 & 0 \\ 0 & 1 & 0 \\ k'_x & 0 & 1 \end{pmatrix}$$

where  $u_m$  is the location difference for these two middle vertical epipolar lines. The matrix  $\mathbf{A}_3$  of the above form can be used to align all the matching vertical epipolar lines. The  $w_x$  and  $k'_x$  will be obtained using the similar method as for  $\mathbf{A}_2$ . Here the  $w_x$  and  $k'_x$  values are obtained by:

$$\begin{cases} w_x = \frac{(G_2 - G_1)H_1H_2}{(H_2 - H_1)G_1G_2} \\ k'_x = \frac{H_1G_2 - H_2G_1}{(H_2 - H_1)G_1G_2} \end{cases}$$

where the values of  $H_1$ ,  $H_2$  (in left image),  $G_1$  and  $G_2$  (in top image) are the location values of vertical epipolar lines on the  $x$ -axis. The alignment transformation  $\mathbf{A}_2\mathbf{T}_v$  corresponds to  $\mathbf{P}_{1x}$ .  $\mathbf{K}_y$  needs to be applied to the right image to correspond to the transformation  $\mathbf{P}_1$  for the reference image. This is possible because of the special form of  $\mathbf{K}_x$  and  $\mathbf{K}_y$  which makes the following equation hold

$$\mathbf{K}_x\mathbf{K}_y = \mathbf{K}_y\mathbf{K}_x$$

We use this special property of the projection matrix  $\mathbf{K}$  to rectify the image triplets.

The alignment transformation  $\mathbf{A}_3\mathbf{T}_u$  for the top image corresponds to  $\mathbf{P}_{1y} = \mathbf{K}_y\mathbf{SR}_1\mathbf{T}$ . Similarly  $\mathbf{K}_x$  needs to be



### 3. EXPERIMENTAL RESULTS

This section shows some of the trinocular stereo image rectification results obtained using our new method described in previous sections. A variety of real images have been tested. For each triplet of images we have the left-right pair of images and left-top pair of images. The fundamental matrix for each pair of images can be obtained using the methods proposed in [24, 25]. When resampling the input images for rectification, bilinear interpolation can be used.

Figure 1(a,b,c) shows an input image triplet (top image in (a), left/reference image in (b), and right image in (c)) with some epipolar lines overlaid. In the reference image, there are two sets of epipolar lines passing through two epipoles. Figure 1(d,e,f) shows the rectified images with overlaid horizontal and vertical epipolar lines. The matching horizontal epipolar lines are in Figure 1(e,f) while the matching vertical epipolar lines are in Figure 1(d,e). In Figure 1(e), the two sets of epipolar lines are orthogonal to each other and parallel to the image axes.

The rectification process has been applied to a dozen of image triplets, and very convincing results are obtained. Figure 2(a,b,c) shows the rectification result of another example image triplet. The matching epipolar lines become horizontal or vertical.

The running time of our rectification process is about 0.395 s for a  $756 \times 504$  pixel image on a Linux PC with a 3.00GHz Intel Core2 Duo CPU using the C language. The majority of the CPU time is on the actual image resampling process which takes about 0.384 s. The process for obtaining the rectification matrices only takes about 11.7 ms.

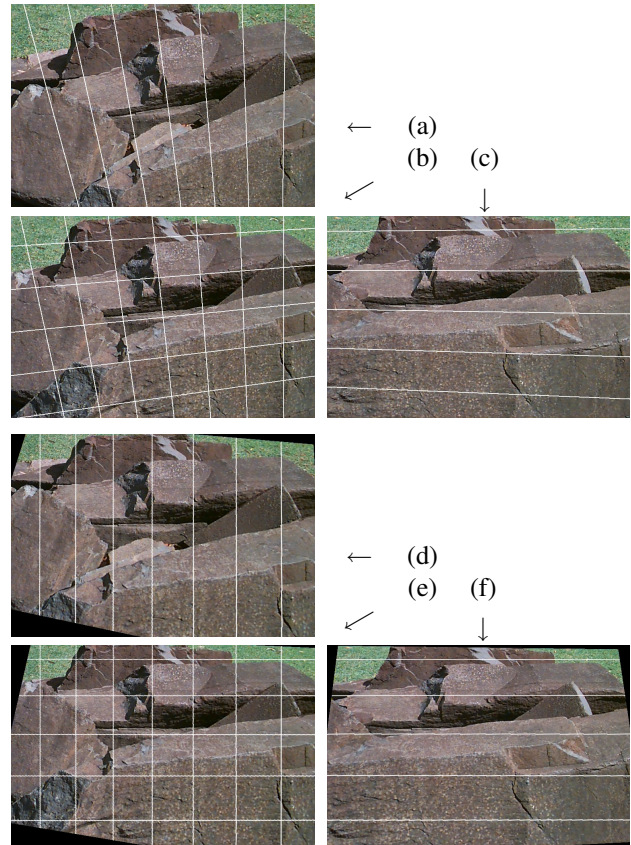
### 4. ONLINE DEMO

An online web demo for trinocular stereo image rectification using our method is available at the following address. Interested readers can try their own images.

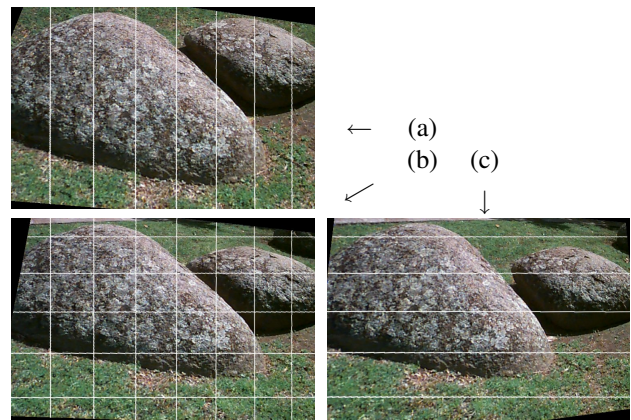
[vision-cdc.csiro.au/rectify3v](http://vision-cdc.csiro.au/rectify3v)

### 5. CONCLUSIONS

In this paper, a new method for automatically rectifying uncalibrated trinocular stereo images has been presented. The rectification matrices applied to the original trinocular images are obtained just based on the epipolar geometries between image pairs, i.e., only using the fundamental matrices. The matching points that may have been used initially for estimating the fundamental matrix are not used during the rectification process. Therefore, the algorithm does not depend on the matching points in the images. There is no iterative parameter optimization process in our method. That is, our method is in closed-form and just uses the fundamental matrices. Real images have been tested and the results validate our new method.



**Fig. 1.** Trinocular image rectification. (a,b,c) Original top, left, and right images with epipolar lines overlaid. (d,e,f) Rectified images with epipolar lines being horizontal or vertical.



**Fig. 2.** (a,b,c) Rectified trinocular images for another triplet. Epipolar lines are becoming horizontal (left-right) or vertical (left-top).

Future work should include more quantitative evaluations of different rectification methods.

## 6. REFERENCES

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